OPTIMUM DESIGN OF PRESTRESSED CONCRETE RAILWAY SLEEPERS

A Thesis Submitted

in Partial Fulfillment of the Requirements

for the Degree of

Master of Technology

by

Maloy Kumar Singha

to the

DEPARTMENT OF CIVIL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

March 1997

CERTIFICATE

Certified that the work contained in the thesis entitled "*OPTIMUM DESIGN OF PRESTRESSED CONCRETE RAILWAY SLEEPERS*", by "*Maloy Kumar Singha*", has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

(Dr. A.S.R. Sai)

Department of Civil Engineering,
Indian Institute of Technology,
Kanpur.

Dedicated
to my
parents

ABSTRACT

Design of prestressed concrete railway sleeper is one of the main problems of railway engineering and when its density is 1660/km in railways its minimum cost design is very much essential.

One standard model for prestressed concrete railway sleeper has been taken with variable dimensions, which can represent any possible shape of it. Detailed analysis procedure for getting bending moment, shear force and stresses in the sleeper is presented. Taking the total cost of the sleeper as objective function and stresses not exceeding the code recomended limiting values as constraints, the problem has been mathematically formulated. The problem has been solved by using genetic algorithm. The effects of impact factor of load and modulus of foundation have been studied. Results have been compared with the existing design

ACKNOWLEDGEMENTS

It gives me profound pleasure to express my deep gratitude to my thesis supervisor, Dr. A.S.R. Sai. Professor of civil engineering for his invaluable guidance and wise counsel. He has always been a constant source of encouragement for me.

I wish to place on record my sincere thanks to my teachers who taught me various courses at IIT kanpur. I also wish to appreciate the company of my friends who made my stay at IIT kanpur memorable.

Maloy

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Chapter 1

INTRODUCTION

1.1 Railway sleepers :

Sleepers in railway track perform two important functions (i) hold the track to guage (ii) transmit and distribute the overcoming loads to the ballast underneath.

In the historical past, sleepers for railway track consisted of slabs of stones or longitudinal timbers laid continuously under the rails. With the evolution of better design of rails, it is not necessary to give a continuous support to the rails. Intermittent supports, with a positive means of holding the rails were found to be more advantageous. This led to the adoption of cross sleepers, which were first introduced in Britain in 1835 and now widely used. The requirements of these sleepers are,

(i) good hold over track geometry,

(ii) fitness for modern track structure i.e, heavy weight to lend stability to track, long retentability to packing to avoid frequent attention to track,

(iii) ability to absorb energy and vibration i.e, sleepers should be able to absorb the impact energy and damp the vibration to a considerable extent and transmit

the balance to the ballast for it to take care of,

(iv) easy to manufacture, transport and laying,

(v) antisabotage i.e, the fastening should provide safety against theft and sabotage,

(vi) long service life,

(vii) low maintenance and

(viii) economy.

For many decades, wood was the only material used by world railway system for the manufacture of the sleepers. Scarcity of good quality timber because of ecological requirements, its rising price, susceptibility to fire and comparatively short life span led to the development of other types of sleepers. Metal sleepers are used as an alternative material in many places. Two types of metal sleeper are used - cast iron sleeper and steel sleeper. After that concrete sleepers appeared on the railway scenario as an alternative to the wooden and metal sleepers. Mass scale production of concrete sleepers in India was taken up in 1967-68 (1).

The majority of concrete sleepers now produced in India are monoblock prestressed concrete sleepers, though trial production of two block reinforced concrete sleepers is going on. The advantages of concrete sleeper are its heavy weight, good longitudinal and lateral resistance, long service life of around 50 years as compared to 16-20 years for wooden sleeper and better track geometry. The manufacture of prestressed concrete railway sleepers, their transport, laying and maintenance demand superior technology, which is to be improved.

1.2 Assessment of loads on railway sleepers

When train rolls on the rails much more load than the static load of the wheels comes on the rails and consequently on the sleepers. The major factors, which cause this load variation are, subgrade characteristic, axle spacing, train speed, surface alignment of track, condition of rails and joints, roughness of rail surface, rail stiffness, condition of wheels, vehicle condition, train operating system and sleeper spacing.

Many models have been devised and analyzed to represent the real situation. The models can be track model, vehicle model or a combination of the two. Different railways in the world are measuring the actual forces which are coming on sleepers and constructing models to explain the real situation. Sato and Satoh (2) have measured the force coming on sleepers under the train running at high speed in Japan and investigated the causes of load increase and prepared a model representing the track irregularities. Prud'homme (3) of French national railway has also done the same thing. Some vehicle track combined model has been done by Birmann (4) in West Germany. One track modeling has been done by Ahlbeck et al (5) in the United States of America. A non-linear analysis has been done by Raymond et al (6) in Canada.

It is impossible to construct a single mathematical model which can be universally addressed to all aspects of train-track interaction. The mechanics involved are so complex, because of their non-linear nature and large number of parameters involved, that one must face the following alternatives - either the model is simple and easy to analyze but is far from reality or the model is so complex that it is unmanageable to handle and in the obtained results, the important parameters may hide behind the less important ones.

So the design force on concrete sleeper will depend upon the track, train condition and train speed. It will be different for different countries and different railways. It has to be decided experimentally and locally. In India, Research Designs and Standards Organisation (RDSO) is also doing different kinds of experiments (7). Constructing models of sleeper they are conducting experiments applying artificial vibrational force of different frequency and different magnitude, there by trying to model the field condition.

American railway has recommended (8) the impact factor as 1.5 . So the design load considering the dynamic effect is an equivalent static load and is 1.5 times the static wheel load. Indian railways adopted the loading standard, followed by German railways with a slight modification. Singh and Rao (9) have recommended the the design force as speed factor times the static wheel load, where speed factor is a constant to be assessed from the track parameters.

1.3 Why optimization of railway sleepers

The optimization of a structural design is to achieve economy besides safety of the structure. Since the civil engineering structures are usually cost intensive, any saving of cost, however little it may be, will lead to appreciable reduction of the over all cost of the structure. In railway track, where the number of concrete sleepers used is 1660/km, it is very essential to achieve a minimum cost design which can withstand the maximum load expected to come on it during its life time. The

optimization seeks to improve the performance towards some optimal point. Apart from the economic advantage gained, there is the satisfaction of producing the best design possible.

There is another advantage of using optimum design concepts. The analysis and design of a structural system are mutually coupled processes. The bending moment and shear force coming on prestressed concrete sleepers depend on its dimensions, where the nature and magnitudes of the bending moment and shear force decide its dimensions. Hence in the conventional design, a trial section is assumed and analyzed and the results are compared with the permissible and preassigned limiting values and modified if required. The process is repeated till all the constraints are satisfied. Hence the conventional design process is iterative, and time consuming, but it does not give much attention towards the minimization of the cost of the structure. Even with the aid of computers, a trial and error approach has to be followed in the absence of a particular direction to approach a solution. Where as the optimization techniques seek to find the optimum solution by proceeding systematically in a definite direction, hence provide a means of speeding up the design process apart from finding a feasible and economic solution.

1.4 Structural optimization via Genetic algorithms

The Genetic algorithms (GAs) are recent addition to the methods of optimization. It is a computerized search and optimization algorithm based on the mechanics of natural genetics and natural selection. Professor John Holland of the University of Michigan first developed the concept of Genetic algorithm in the mid sixties. There after a number of his students and others have contributed in developing the field.

The success of GAs owes much to the work of David F. Goldberg, a professor at the University of Illinois. Goldberg and Samtani (10) were the first to apply GA for the optimization of a 10 bar truss and reported that the convergence is not dissimilar to that observed in other studies. Deb (11) applied GA for optimum design of welded beam structure and found comparable results. Rajeev and Krishnamurthy (12) applied GA for the discrete optimization of structures.

The traditional optimization methods can be broadly grouped into two categories as direct search and descent methods. The direct search method requires only objective function value and it performs local search around the current point to find a better new point. But it takes very large time to converge to the final solution. Descent methods need first and sometimes second order derivatives of the function in addition to the function value. Their drawback is that they are suitable for problems having smooth, continuous and unimodal objective function. These problems can be overcome by GAs, since they are not limited by restrictive assumptions about the search space regarding the continuity or existence of derivatives. Any real world problem can be conveniently tackled by GAs. The ability of GAs to handle discrete variables makes it a versatile tool for structural optimization.

1.5 Organisation:- scope of the thesis

The optimum design of prestressed concrete railway sleeper has two different sides;

(a) optimum shape and size to be able to withstand the maximum load, expected to come on it during its life time and provide stability,

(b) optimum selection of material that involves quality of the materials used,

their strength, durability and cost.

For the present work the first one has been selected i.e, optimum shape and size.

To design a practical structure it is desirable to go as much analytical as possible. But to handle the natural situation is so tough that sometimes some assumptions have to be made considering the validity of those with respect to the information and experience available to us. Here it is assumed that the dynamic effect of the train is equal to impact factor times the static wheel load(22.5t) as described in section 1.2. Also it is assumed that the materials used for prestressed concrete railway sleepers - the concrete and steel have the strength as specified in Indian codes. Also it is assumed that the sleepers are resting on ballast cushion which provide uniform modulus of foundation.

One standard model for prestressed concrete railway sleeper has been taken with variable dimensions, which can represent any possible shape of it. Detailed analysis procedure for getting bending moment, shear force and stresses in the sleeper is presented in chapter 2. Taking the total cost of the sleeper as objective function and stresses not exceeding the code recommended limiting values as constraints, the problem has been mathematically formulated in chapter 3. The problem has been solved by using genetic algorithm. The effects of impact factor and modulus of foundation have been studied. Results have been compared with the existing design and given in chapter 4.

Chapter 2

ANALYSIS OF SLEEPERS

This chapter presents the analysis procedure of prestressed concrete railway sleepers. First analytical approach and then numerical techniques of analysis are discussed. Both one dimensional beam analysis and two dimensional finite element analysis are presented.

2.1 Simple beam bending approach

The sleeper can be considered as a simple beam resting on elastic foundation with constant modulus of foundation as shown in Fig 2.1

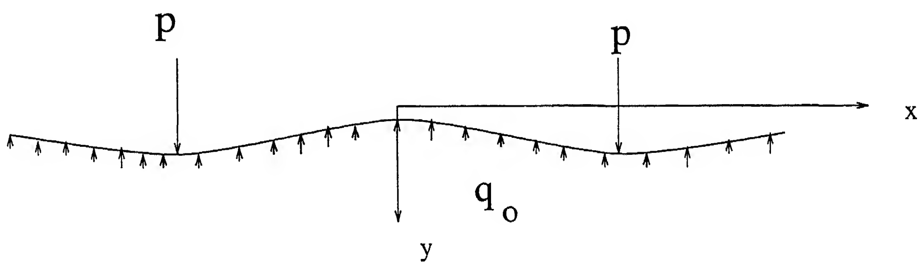

Fig. 2.1 Beam on elastic foundation

$q_0 = k_o b_x y = k_x y$

where

b_x = breadth of the beam at section x,

k_0 = modulus of foundation, and

$k_x = k_o b_x$.

From elementary strength of material theory we know

$$EI_x \frac{d^2y}{dx^2} = -M_x \qquad \text{and} \qquad \frac{d^2M_x}{dx^2} = k_x y$$

where

I_x = moment of inertia of the beam section at distance x, and

M_x = bending moment at distance x.

Then combining these two equations one gets

$$\frac{d^2}{dx^2}[EI_x \frac{d^2y}{dx^2}] + k_x y = 0 \tag{2.1}$$

This is the differential equation for the deflection curve of the beam resting on elastic foundation.

Boundary conditions:-

at one end moment=0, shear=0

at the middle slope=0, shear=0

(due to symmetry)

When the expressions of I_x and k_x are simple then one can solve this differential equation analytically. But if the expressions of I_x and k_x are very complex then it is very difficult to solve this differential equation analytically. Then one must go for numerical techniques for analysing the sleeper.

2.2 Numerical analysis:-

One way to solve the problem is to solve the differential equation (2.1) with those boundary conditions numerically. Another way is to divide the system into finite number of simple parts of finite dimension and finding out a stress and displacement function, that satisfies the differential equation of equilibrium, the stress - strain relationships and compatibility conditions at any point within the element and element boundaries. Then using the continuity at the element boundaries the total system can be formulated and solved.

When number of elements (n) considered is infinitely large (limit n tending to infinity) then the approach is called the classical approach but when number of elements is finite then the approach is called finite element analysis.

2.2.1 One dimensional analysis:-

One can divide the sleeper into a finite number of beams along the length of the sleeper and then he can consider the cross section of each beam as constant. The joints of the two element beams can be considered as global nodes.

Element analysis:-

Now I_x and k_x are constant. General solution of equation (2.1) with constant I_x (I_x = I) and k_x (k_x = k)is

$$y = e^{\lambda x}[c_1 cos\lambda x + c_2 sin\lambda x] + e^{-\lambda x}[c_3 cos\lambda x + c_4 sin\lambda x] \tag{2.2}$$

where

$$\lambda = \sqrt[4]{\frac{k}{4EI}}$$

where c_1 , c_2 , c_3 and c_4 are constants to be determined from the boundary conditions.

Let at one end (x=0) the initial conditions be deflection y = y_0, slope θ = θ_0, moment M = M_0 and shear V = V_0 then putting these boundary conditions one gets

$$y_x = y_0F_1(\lambda x) + \frac{\theta_0F_2(\lambda x)}{\lambda} - \frac{M_0F_3(\lambda x)}{\lambda^2EI} - \frac{V_0F_4(\lambda x)}{\lambda^3EI} \tag{2.3}$$

$$\theta_x = \theta_0F_1(\lambda x) - \frac{M_0F_2(\lambda x)}{\lambda EI} - \frac{V_0F_3(\lambda x)}{\lambda^2EI} - 4\lambda y_0F_4(\lambda x) \tag{2.4}$$

$$M_x = M_0F_1(\lambda x) + \frac{V_0F_2(\lambda x)}{\lambda} + \frac{ky_0F_3(\lambda x)}{\lambda^2} + \frac{k\theta_0F_4(\lambda x)}{\lambda^3} \tag{2.5}$$

$$V_x = V_0F_1(\lambda x) + \frac{ky_0F_2(\lambda x)}{\lambda} + \frac{k\theta_0F_3(\lambda x)}{\lambda^2} - 4\lambda M_0F_4(\lambda x) \tag{2.6}$$

where

$$F_1(\lambda x) = cosh\lambda x cos\lambda x \tag{2.7}$$

$$F_2(\lambda x) = \frac{cosh\lambda x sin\lambda x + sinh\lambda x cos\lambda x}{2} \tag{2.8}$$

$$F_3(\lambda x) = \frac{sinh\lambda x sin\lambda x}{2} \tag{2.9}$$

$$F_4(\lambda x) = \frac{cosh\lambda x sin\lambda x - sinh\lambda x cos\lambda x}{4} \tag{2.10}$$

So at a distance x the deflection, slope, moment and shear can be expressed as

$$\begin{Bmatrix} y_x \\ \theta_x \\ M_x \\ V_x \end{Bmatrix} = \begin{bmatrix} F_1 & \frac{F_2}{\lambda} & -\frac{F_3}{EI\lambda^2} & -\frac{F_4}{EI\lambda^3} \\ -4\lambda F_4 & F_1 & -\frac{F_2}{EI\lambda} & -\frac{F_3}{EI\lambda^2} \\ \frac{kF_3}{\lambda^2} & \frac{kF_4}{\lambda^3} & F_1 & \frac{F_2}{\lambda} \\ \frac{kF_2}{\lambda} & \frac{kF_3}{\lambda^2} & -4\lambda F_4 & F_1 \end{bmatrix} \begin{Bmatrix} y_0 \\ \theta_0 \\ M_0 \\ V_0 \end{Bmatrix} \tag{2.11}$$

or

$$\begin{Bmatrix} y_x \end{Bmatrix} = [F(\lambda x)] \begin{Bmatrix} y_0 \end{Bmatrix} \tag{2.12}$$

<u>Total sleeper as element wise:-</u>

If the length of the element beams are δl and left end initial conditions are $\begin{Bmatrix} y_0 \end{Bmatrix}$ then the right hand side of the first beam element will have boundary conditions $\begin{Bmatrix} y_1 \end{Bmatrix}$, which can be expressed as

$$\begin{Bmatrix} y_1 \end{Bmatrix} = [F(\lambda_1 \delta l)] \begin{Bmatrix} y_0 \end{Bmatrix} = [F]_1 \begin{Bmatrix} y_0 \end{Bmatrix} \tag{2.13}$$

Similarly the boundary conditions at the end of second element are

$$\begin{Bmatrix} y_2 \end{Bmatrix} = [F]_2 \begin{Bmatrix} y_1 \end{Bmatrix} = [F]_1 [F]_2 \begin{Bmatrix} y_0 \end{Bmatrix} \tag{2.14}$$

Similarly at the right end of the ith element i.e, at the ith node one have

$$\begin{Bmatrix} y_i \end{Bmatrix} = [F]_i \begin{Bmatrix} y_{i-1} \end{Bmatrix} \tag{2.15}$$

If at (i-1)th node some force P is externally applied then the shear force will be reduced by P, i.e, V_{i-1} will change to (V_{i-1} - P).

Similarly at the right end of the nth element i.e, at the right end of the sleeper the boundary conditions will be y_n, θ_n, M_n and V_n and they can be expressed as

$$\begin{Bmatrix} y_n \end{Bmatrix} = [F]_1 [F]_2 ...... [F]_n \begin{Bmatrix} y_0 \end{Bmatrix} \tag{2.16}$$

So starting from one end one can proceed element wise towards the other end expressing the deflection, slope, moment and shear forces in terms of initial conditions at that end. Finally he can reach at the other end by equation (2.16). Now out of eight initial conditions y_0, θ_0, M_0, V_0 at the left end and y_n, θ_n, M_n and V_n at the right end any four will be known from boundary conditions. Remaining four can be solved using equation (2.16). Once these initial conditions are known, one can calculate the deflection, slope, bending moment and shear force through out the length of the beam using equations (2.12) to (2.15)

2.2.2 Two dimensional Finite Element analysis

This is a plane stress problem. The differential of equilibrium is

$$\begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.17}$$

The stress -strain relation is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} \tag{2.18}$$

or
$$\{\sigma\} = [\mathrm{D}] \{\epsilon\}$$

The strain - displacement relation is

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{2.19}$$

or

$$\left\{\epsilon\right\} = [T] \begin{Bmatrix} u \\ v \end{Bmatrix}$$

Total sleeper can be divided into several rectangular or triangular elements.

Element analysis:-

Now in this case the elements cannot be solved analytically for displacement or stress field. Because analytical solution of equation (2.17) to (2.19) is not possible. Some key points inside the element can be considered as nodes. Some displacement shape function can be assumed in terms of the nodal displacements within the element as,

$$u = \sum_{j=1}^{n} u_j \psi_j \qquad \text{and} \qquad v = \sum_{j=1}^{n} v_j \psi_j$$

where

u = displacement in x direction inside the element,

v = displacement in y direction inside the element,

u_j = x displacement of jth node,

v_j = y displacement of jth node,

n = number of nodes inside the element, and

ψ = Lagrange interpolation function of degree (n-1).

So the displacement vector inside the element can be written as

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} \psi_1 & 0 & \psi_2 & 0 & . & . & \psi_n & 0 \\ 0 & \psi_1 & 0 & \psi_2 & . & . & 0 & \psi_n \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ .. \\ .. \\ .. \\ u_n \\ v_n \end{Bmatrix} \tag{2.20}$$

or

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [\psi]\begin{Bmatrix} u^e \end{Bmatrix} \tag{2.21}$$

Hence

$$\begin{Bmatrix} \epsilon \end{Bmatrix} = [B]\begin{Bmatrix} u^e \end{Bmatrix} \tag{2.22}$$

where

$$[\mathrm{B}] = [\mathrm{T}]\ [\psi]$$

Now using the principle of virtual displacement we get (13)

$$[K^e]\begin{Bmatrix} u^e \end{Bmatrix} = \begin{Bmatrix} f^e \end{Bmatrix} \tag{2.23}$$

where

$\begin{Bmatrix} f^e \end{Bmatrix}$ = nodal force vector,

$\begin{Bmatrix} u^e \end{Bmatrix}$ = nodal displacement vector,

$[K^e]$ = element stiffness matrix, and

$$[K^e] = h_e \int_S [B^e]^T [D][B^e] dx dy \tag{2.24}$$

Total sleeper:-

From element stiffness matrix the global stiffness matrix can be formed. Now displacement boundary conditions in terms of nodal displacements can be used to modify the global stiffness matrix and finally from that the nodal displacements and stresses can be found out.

Chapter 3

PROBLEM FORMULATION

This chapter presents a mathematical model of prestressed concrete railway sleeper. The design variables are selected. The cost of one sleeper is taken as objective function. The design constraints are formulated and expressed in suitable form. Finally the constrained optimization problem is transformed into unconstrained optimization problem by using penalty function.

3.1 Problem description and presentation of model

Design of prestressed concrete railway sleeper needs its proper shape and size so that it is just sufficient to withstand the design load. But the bending moment and shear force coming on the sleeper depend on its dimensions. So at first one model has to be taken with variable dimensions and then an optimization algorithm can be used to find out the values of those variable dimensions, such that the cost of the sleeper is minimum.

It is clear that a rectangular or a trapezoidal section or a combination of both, along the length of the sleeper will be suitable. The shape of existing concrete sleepers are consistent with this idea. The length of the sleeper and the length of

various types of sections are also to be decided. So a model of prestressed concrete sleeper with variable dimensions has been taken, such that it can represent any possible shape of the sleeper and is shown Fig. 3.1

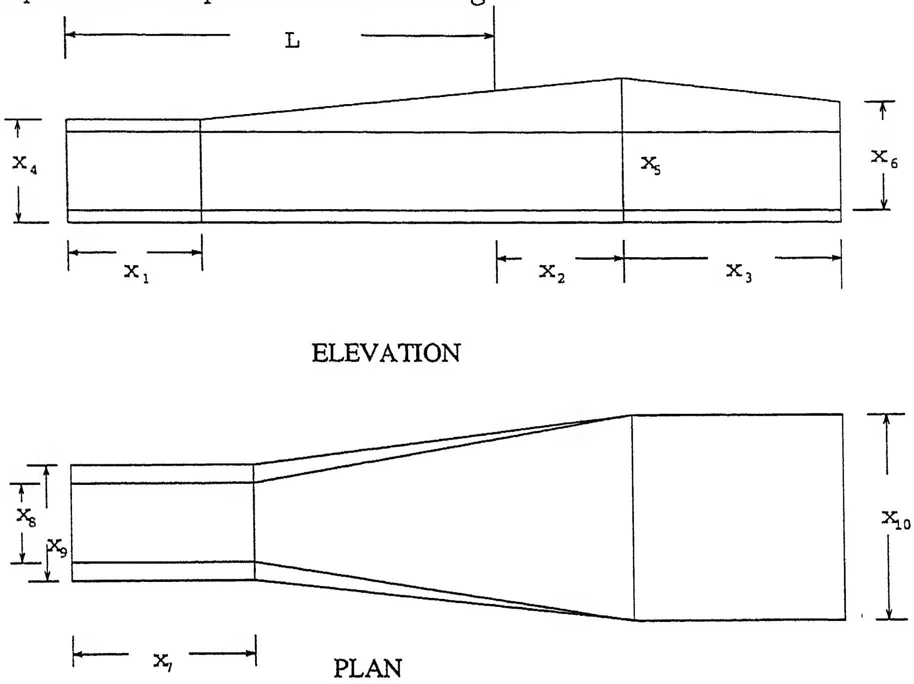

Fig. 3.1 Model of the sleeper

Since the sleeper is symmetrical, only half of it is shown in the figure. The guage length is 2L. The shape of the sleeper is expressed in 10 variables (x_1 to x_{10}).

x_{11} = percentage of bottom reinforcement.

x_{12} = percentage of top reinforcement.

Effective clear cover to reinforcement is 40 mm

Now the optimum values of these 12 design variables (x_1 to x_{12}) are to be found out so that the cost of the sleeper is minimum and at the same time it can withstand the design load.

3.2 Objective function

Volume of the sleeper $V = x_1 x_4 (x_8 + x_9) + x_3 x_{10} (x_5 + x_6)$

$$+(x_7 - x_1)(x_8 + x_9)[x_4 + \frac{(x_5-x_4)(x_7-x_1)}{2(L+x_2-x_1)}]$$

$$+(L+x_2-x_7)\frac{(x_8+x_9+2x_{10})}{2}[x_5 - \frac{(L+x_2-x_7)(x_5-x_4)}{2(L+x_2-x_1)}] \quad (3.1)$$

Volume of concrete V_c = [1- (x_{11} + x_{12})] V

Volume of steel V_s = (x_{11}+ x_{12}) V

Then total cost per sleeper= $C_0 + C_c V_c + C_s V_s$

$=C_0 + C_c\ (V_c + \mathrm{C} V_s)$

where

C_c = cost of concrete per unit volume,

C_s = cost of steel per unit volume,

C_0 = some constant cost, and

C = ratio of cost of steel and concrete=C_s/C_c.

So the objective function to be minimized is

$$f(x)=C_0 + C_c V[1 - x_{11} - x_{12} + C(x_{11} + x_{12})] \quad (3.2)$$

3.3 Design constraints

Design constraints are stresses, which should not go beyond the allowable limits. So the calculated bending moment and shear force due to design load at any section should not exceed the moment capacity and maximum shear capacity of that section. Further for dynamic load a load factor of safety of 2.5 should be used.

So

So

$$M_{r_i} \geq 2.5\ M_{l_i} \qquad \text{i=1,10} \qquad \text{at 10 selected points.}$$

$$V_r \geq 2.5V_l \qquad \text{at the point of application of load.}$$

where

M_r = moment of resistance of the section,

M_l = moment due to load,

V_r = shear resistance, and

V_l = shear force due to load.

For dynamic load the section should be over reinforced. So

$$x_{11} \geq \frac{14.49}{15+0.0435f_p}\ \frac{f_{ck}}{f_p}$$

Again the reinforcement percentages should have a limiting value of $0.24\frac{f_{ck}}{f_p}$,

where

f_{ck} = chacteristic strength of concrete and

f_p = chacteristic strength of steel.

3.4 Problem formulation

$$\min\ f(x) = C_0 + C_c V[1 - x_{11} - x_{12} + C(x_{11} + x_{12})]$$

subject to

$$g_i(x) = M_{r_i} - 2.5\ M_{l_i} \geq 0 \qquad i = 1\ ,\ 10$$

$$g_{11}(x) = V_r - 2.5V_l \geq 0$$

$$g_{12}(x) = x_{11} - \frac{14.49}{15+0.0435f_p}\frac{f_{ck}}{f_p} \geq 0$$

$$g_{13}(x) = 0.24\frac{f_{ck}}{f_p} - x_{11} \geq 0$$

$$g_{14}(x) = 0.24\frac{f_{ck}}{f_p} - x_{12} \geq 0$$

3.5 Transformation of the optimization problem

This is a 12 variable optimization problem with 14 constraints. To handle the constraints the best way is to use penalty function method and there by transforming the optimization problem into an unconstrained optimization problem. Here one can add bracket operator penalty function P(x) = R $< g_i(x) >^2$ to the objective function, where $< g_i(x) > = g_i(x)$ when $g_i(x)$ is negative; zero otherwise and R is the penalty parameter. Since this operator adds positive value to the objective function at infeasible points and the problem is a minimization problem it forces the minimum point to lie in the feasible region. Hence the optimization problem becomes

$$\min$$

$$f(x) = C_0 + C_c V[1 - x_{11} - x_{12} + C(x_{11} + x_{12})] + 100.0 \sum_{i=1}^{14} < g_i(x) >^2 \qquad (3.3)$$

This optimization problem now can be handled by Genetic algorithm described in appendix.

Chapter 4

RESULTS AND DISCUSSION

4.1 Results

The problem formulation, presented up to now has been coded in Fortran 77 code in a digital computer. Genetic Algorithm is then used to find out the optimum solution of the problem. Attempt has been made to reach the global optimum solution by using different sets of GA parameters. With the following problem input data, the following result has been obtained.

Problem input data:-

Grade of concrete = M45

Grade of steel = Fe 1600

Guage (B.G.) length L = 175 cm

Cost of concrete C_c = 2600 Rs/cm^3

Cost ratio C =$\frac{C_s}{C_c}$= 160

Constant cost C_0 = 40 Rs

Modulus of foundation K_0 = 2.6 kN/cm^2/cm

Impact factor of load = 1.5

Coding of variables

Each variable can take only discrete values within their bounds as shown in Table 4.1

Table 4.1 Bounds on variables

	x_1 mm	x_2 mm	x_3 mm	x_4 mm	x_5 mm	x_6 mm	x_7 mm	x_8 mm	x_9 mm	x_{10} mm	x_{11}	x_{12}
minimum	100	180	100	180	225	160	200	150	180	220	0.45	0.45
maximum	410	255	175	255	300	235	510	225	255	295	0.60	0.60
increment	10	5	5	5	5	5	10	5	5	5	0.01	0.01

Solution

The best solution vector found using genetic algorithm is

$(350, 180, 100, 220, 270, 170, 440, 160, 220, 270, 0.472, 0.453)^T$

Total cost Rs. 792.3 volume = 116672.6 cm^3

Table 4.2 shows the best point found at some intermediate generation with a population size of 1000, cross over probability of 0.735, and mutation probability of 0.0135.

Table 4.2 Intermediate results

generation number	The best design vector in mm	fitness
1	$(245, 180, 115, 245, 285, 190, 410, 160, 235, 255, 0.47, 0.46)^T$	0.00101
16	$(220, 180, 100, 210, 265, 190, 335, 155, 235, 280, 0.47, 0.46)^T$	0.00110
18	$(310, 185, 100, 195, 280, 195, 435, 155, 245, 260, 0.50, 0.46)^T$	0.00114
109	$(220, 185, 100, 240, 265, 165, 395, 160, 200, 295, 0.49, 0.49)^T$	0.00115
138	$(245, 180, 100, 200, 285, 170, 430, 160, 245, 260, 0.49, 0.47)^T$	0.00120
191	$(350, 180, 100, 220, 270, 170, 440, 160, 220, 270, 0.47, 0.46)^T$	0.00125

There are two main parameters, which decide the design. They are modulus of foundation (k_0) and impact factor (P_i) of load. The effect of both is studied one by one.

4.1.1 Effect of impact factor (P_i) of load on design

With constant modulus of foundation $K_0 = 2.6$ kN/cm^2/cm, for different impact factors of load the results have been given in Table 4.3

Table 4.3 Variation of design with impact factor of load.

P_i	x_1 cm	x_2 cm	x_3 cm	x_4 cm	x_5 cm	x_6 cm	x_7 cm	x_8 cm	x_9 cm	x_{10} cm	A_b mm^2	A_t mm^2	Cost Rs	Vol. m^3
1.3	28	18	10	19	26	17	30	18	20	22	204	196	722	0.100
1.4	31	18	10	21	26.5	17	43	16	22	24	225	214	758	0.110
1.45	32	18	10	21	27	17	46	16	23	25	230	219	772	0.113
1.5	35	18	10	22	27	17	44	16	22	27	238	230	792	0.116
1.55	36	18	10	22	27.5	17	37	16	23	27	247	234	824	0.121
1.6	39	18	10	22	28.5	18	42	17	25	26	256	240	851	0.126

where

A_b = bottom reinforcement, and

A_t = top reinforcement

Variation of optimum values of x_4, x_5, x_{10}, and total cost with impact factor of load have been plotted in Fig 4.1 to Fig 4.4 .

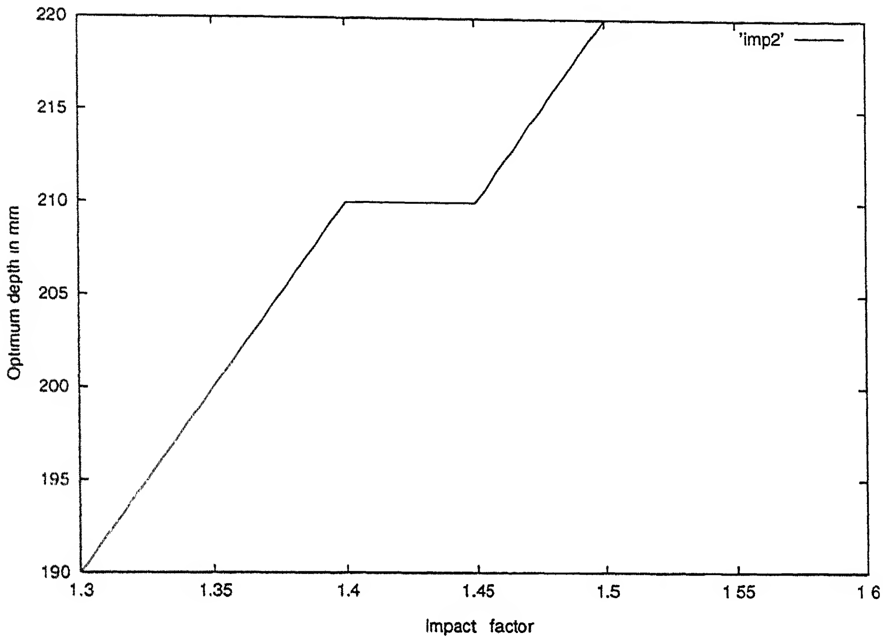

Fig 4.1 Variation of optimum depth x_4 with impact factor

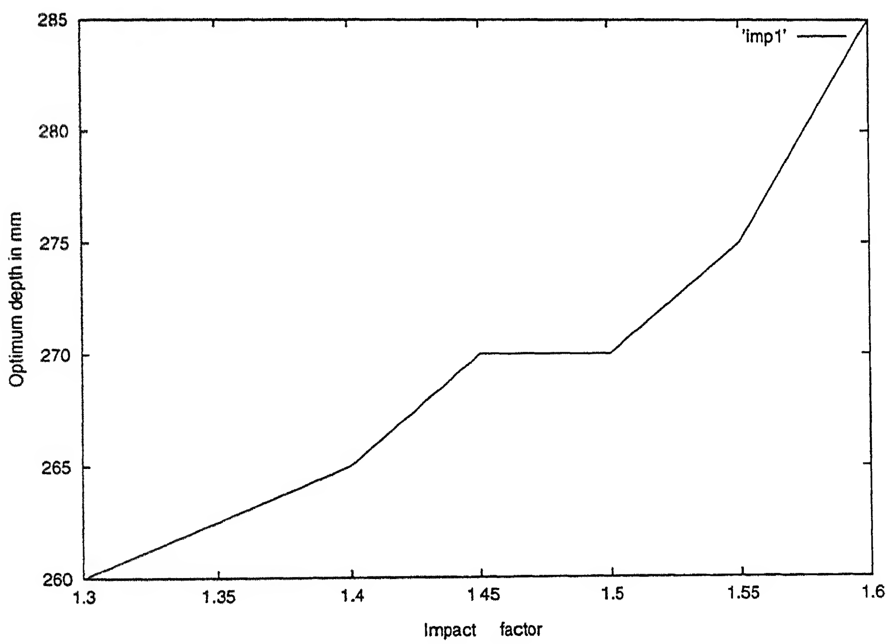

Fig 4.2 Variation of optimum depth x_5 with impact factor

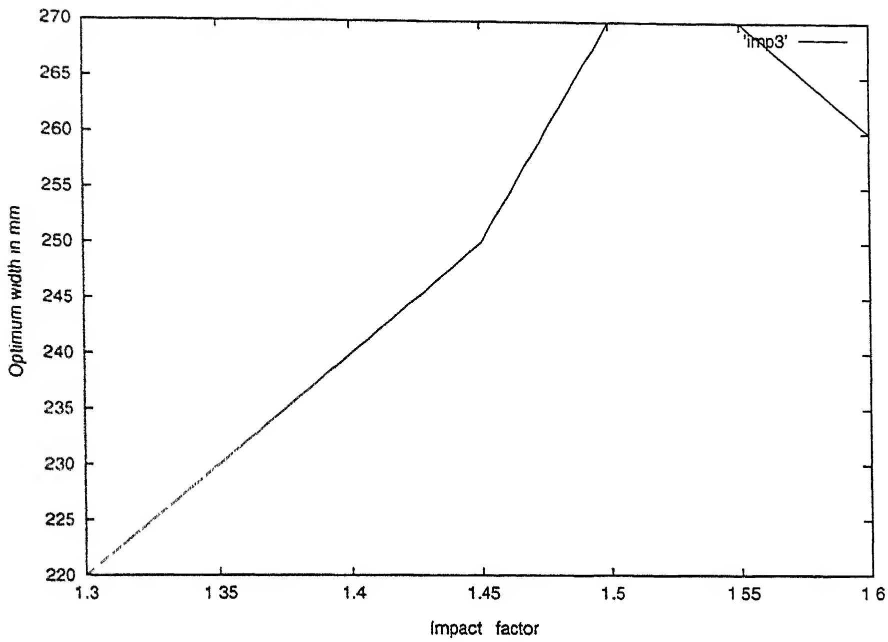

Fig 4.3 Variation of width x_{10} with impact factor

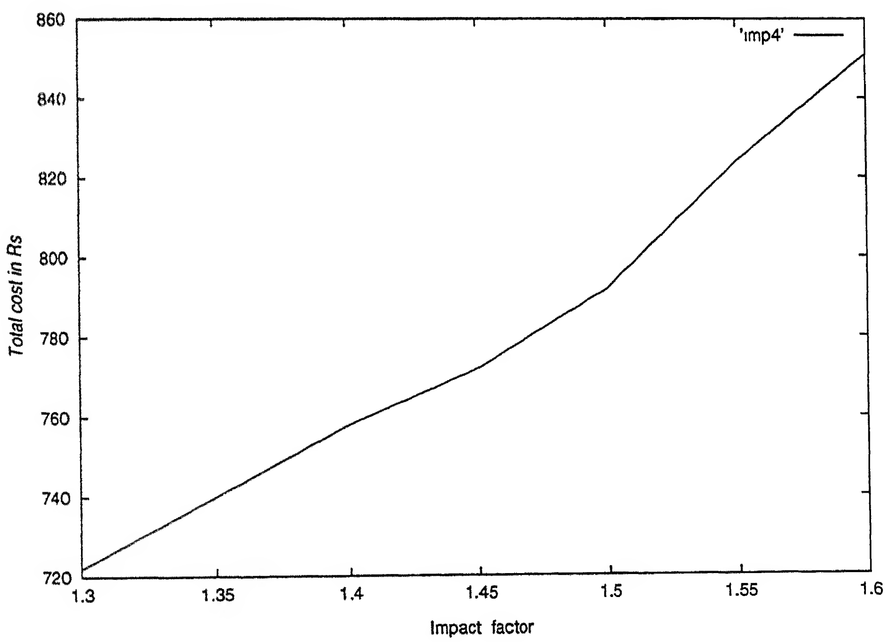

Fig 4.4 Relation of impact factor with total cost of the sleeper

4.1.2 Effect of modulus of foundation

A study of the effect of modulus of foundation on the sleeper is shown here. For the analysis purpose the design obtained using modulus of foundation equal to 2.6 kN/cm^2/cm, and impact factor equal to 1.5 has been taken. The bending moment (B.M.) at the center of the sleeper and at the point of application of the load are shown in Table 4.4 for different moduli of foundation.

Table 4.4 Variation of bending moment with modulus of foundation.

Modulus of foundation kN/cm^2/cm	B.M. at center kN-m	B.M. at point of application of load. kN-m
2.00	-18.8	25.0
2.10	-18.1	25.1
2.20	-17.5	25.1
2.30	-16.9	25.2
2.40	-16.3	25.3
2.50	-15.8	25.3
2.60	-15.4	25.4
2.70	-14.9	25.4
2.80	-14.5	25.4
2.90	-14.1	25.5
3.00	-13.7	25.5
3.10	-13.3	25.5
3.20	-13.0	25.6
3.30	-12.7	25.6

So at lower modulus of foundation bending moment at the center of the sleeper is coming more and hence higher section is needed.

4.1.3 Effect of modulus of foundation (K_0) on design

For impact factor 1.5 results are shown for different moduli of foundation in Table 4.5

Table 4.5 Variation of design with modulus of foundation.

K_0	x_1 cm	x_2 cm	x_3 cm	x_4 cm	x_5 cm	x_6 cm	x_7 cm	x_8 cm	x_9 cm	x_{10} cm	A_b mm^2	A_t mm^2	Cost Rs	Vol. m^3
2.2	37	18	10	22	25	17	44	16	25	29	252	240	848	0.124
2.4	36	18	10	22	26	17	47	16	24	28	249	235	827	0.122
2.6	35	18	10	22	27	17	44	16	22	27	238	230	792	0.117
2.8	37	18	10	21	27	16	41	16	22	25	229	216	767	0.112
3.0	37	18	10	20	28	18	40	17	22	23	226	215	752	0.111

4.1.4 Comparison

The solution obtained is compared with the existing design (R.D.S.O./T-2475), where the solution vector is

$(300, 200, 300, 200, 235, 200, 335170, 220, 270)^T$

cost Rs 918.6, volume= 136489.5 cm^3, steel = 9.7 kg

In present design with modulus of foundation 2.6 kN/cm^2/cm and load impact factor 1.5 the design vector is

$(350, 180, 100, 220, 270, 170, 440, 160, 220, 270, 0.47, 0.45)^T$

cost Rs 792.3, volume= 116672.5 cm^3, steel = 8.3 kg

So the reduction in cost is 13.7 %

4.2 Discussion

4.2.1 Discussion on analysis procedure

The conventional design procedure for a reinforced concrete structure is to determine bending moment and shear force on a member due to load by linear analysis theory after that design that member considering non-linearity in concrete and steel, following the code of practice.

One dimensional analysis presented in section 2.2.1 is simple and the result is converging if the sleeper is divided into 50 parts. On the otherhand in two dimensional FEM analysis, for 3 noded triangular elements at least 100 by 10 nodes are required to converge to such an accurate result. For 6 noded triangular element convergence is some what more, but at least 800 degrees of freedom are required.

4.2.2 Discussion on problem solution

The design depends on two major factors:(a) modulus of foundation (b) impact factor of load. For a constant modulus of foundation, change in impact factor of load from 1.4 to 1.6 changes the total cost of the sleeper from Rs 758 to Rs 851. Similarly when modulus of foundation increases from 2.2 to 3.0 then total cost of the sleeper decreases from Rs 848 to Rs 752 as shown in Table 4.5. More accurate assessment of impact factor and modulus of foundation leads to more accurate design The grades of concrete and steel are input parameters. For any grade of steel and concrete design can be done.

Chapter 5

CONCLUSIONS

5.1 Conclusions

The discrete optimal design of prestressed concrete railway sleeper using genetic algorithm has been presented in the last chapter. Based on the results obtained, the following conclusions may be drawn.

(a) Due to various variables in track and rolling stock, correct assessment of design load has to be based on simple assumption.

(b) Reduction in length of the sleeper is found to be optimum. But the sleeper length should extend beyond the rail seat up to a point, sufficient for casual dispersion of load.

(c) One dimensional beam analysis is found to be more convenient for analysis in this optimization problem.

(d) The following GA parameters are found to give better convergence for this optimization problem.

population size = 1000

probability of crossover = 0.735

probability of mutation = 0.0135

(e) The features of GAs can be effectively utilized to take into account the discreteness of the variables in formulating the optimization problem, which has a lot of practical importance.

(f) Since GA searches the design space within the bounds given for the variables, there is no possibility of variables taking negative values or useless values, which usually happens with the gradient based methods.

(g) Generally GA require more number of function evaluations compared to traditional optimization technique. But the possibility to converge to a local optimum point is removed and that should not be viewed as limitation in the present day computer environment.

5.2 Scope for future work

The concrete sleeper design differs from normal structural design in that the loading and support condition cannot be assessed to a high degree of certainty in the former, as compared to the later. In the present work, for some specific values of modulus of foundation of the supporting ballast cushion and impact factor of load, the optimum designs have been obtained using genetic algorithm. Experiments can be conducted for accurate field determination of the modulus of foundation. Further field study can be made and work can be done towards the assessment of the dynamic effect of the moving train on the railway sleeper.

APPENDIX

INTRODUCTION TO GENETIC ALGORITHMS

The fundamentals of genetic algorithms (GA) are presented here. GAs are computerized search and optimization algorithms based on mechanics of natural genetics and natural selection.

MECHANICS OF GENETIC ALGORITHMS

GAs work with the coding of variables instead of the variables themselves. Coding discretizes the search space even though the function is continuous. Binary coding is most commonly used in GAs application. For example a 4 bit string has its lower and upper value '0000' and '1111' and their decoded value are 0 and 15 respectively. This coding is called string. If for a real variable x the search space is (x^l, x^u) then the strings '0000' and '1111' can be mapped with x^l and x^u respectively, because they have decoded minimum and maximum values. Now any l bit string can be found to represent a point in the search space according to the fixed mapping rule

x=$x^l + (x^u - x^l)/(2^l - 1)$decoded value

With l bit to code each variable only 2^l distinct strings are possible. The accuracy obtainable in that variable discretization is $(x^u - x^l)/(2^l - 1)$. The length of the strings is usually determined according to the solution accuracy desired.The use of these strings are analogous to the natural chromosomes in biological system. Similar to the genes in natural chromosomes, which influences the physical qualities of an individual, the values '0' and '1' in each position of the binary strings determine the co-ordinates of a point in real space. In GA terminology each point is an individual.

GAs work with several number of points at a time collectively known as population. GAs begin with creation of a initial population of individual points at

random within the search space. There after each string is evaluated to find its fitness. In maximization problem fitness of a point is the function value at that point. Minimization problem is transferred to the maximization problem by the transformation

$$f(x)=1.0/(1.0+f(x)).$$

Knowing the fitness values of individuals the population is then operated by three operators -REPRODUCTION, CROSSOVER and MUTATION to create a new population.This procedure of creating new population is called one 'generation' in GA terminology. This creation of new population from old population is continued till the termination criteria is met.

REPRODUCTION

This operator is an artificial version of Darwin's theory of survival of the fittest. The purpose of this operator is to pick up the above average strings from the current population on the basis of their fitness values and to insert multiple copies of them in the mating pool in a probabilistic manner. This operator thus makes highly fit individuals to survive and reproduce and the low fit individuals to die out.

Several selection schemes are in use today. The commonly-used reproduction operator is the proportionate reproduction operator where a string is selected for the mating pool with a probability proportional to its fitness. Thus this operator is expected to copy $f_i/\sum(f_i)$ number of the ith string in the mating pool, where $f_i =$ fitness value of the ith string

CROSSOVER

In reproduction, good strings in a population are probabilistically assigned a larger number of copies and a mating pool is formed, but no new strings are formed.

In crossover operator new strings are created by exchanging the information among the strings of the mating pool. Many crossover operator exits in GA literature. In most crossover operators two strings are picked up at random and some portion of the strings are exchanged between them. A single point crossover operator is performed by randomly choosing a crossing site and by exchanging all bits on the right side of the crossing site as shown

(0 1 1 1 0)	(0 1 1 0 1)
(1 0 1 0 1)	(1 0 1 1 0)
parent strings	child strings

The two strings participating in the crossover operation are known as parent strings and the resulting strings are known as children strings. It is intuitive from this that good substrings from parent strings can be combined to form a better child string if an appropiate site is chosen. Since the better site is not known previously the site is chosen randomly. In the crossover if the child strings are good they will survive in the next reproduction operation otherwise they will die. But it is hopeful that at least some good child string will be produced in this operator.

The total number of strings participating in the crossover operator can be controlled by crossover probability, p_c, which is the ratio of total number of strings selected for crossover operation from the mating pool and the population size.

MUTATION

After crossover mutation operates on the strings for a local search around the current solution. It is used to maintain diversity in the population. The mutation operates 1 to 0 and vice versa with a small mutation probability p_m. The bit-wise mutation is performed bit by bit with a probability p_m.

References

1 Antia, K. F. (1960) "Railway Track", The New Book Company Private Ltd, New Delhi.

2 Kerr, A. D. (1976) "Railroad Track Mechanics and Technology", Pergamon Press, London, pp 63 - 77.

3 Kerr, A. D. (1976) "Railroad Track Mechanics and Technology", Pergamon Press, London, pp 79 - 107.

4 Kerr, A. D. (1976) "Railroad Track Mechanics and Technology", Pergamon Press, London, pp 197 - 222.

5 Kerr, A. D. (1976) "Railroad Track Mechanics and Technology", Pergamon Press, London, pp 239 - 263.

6 Moyar, G. J. , Dilkey, W. D. and Pilkey, B. F. (1978) "Track/Train Dynamics and Design", Pergamon press, Chicago.

7 "Dynamic testing of prestressed concrete railway sleeper of new design to drg. no. T-5475" Report no TM - 2 (1994), Research Design and Standards Organisation, Lucknow.

8 Kerr, A. D. (1976) "Railroad Track Mechanics and Technology", Pergamon Press, London, pp 265 - 281.

9 Singh, K. P. and Rao, C. B. K. (1982) "Design criteria for concrete sleepers", Journal of the Institution of Permanent way Engineers - India, Vol. XI, No 3, pp 1-6.

10 Goldberg, D. E. and Samtani, M. D. (1986) "Engineering optimization via genetic algorithms", Proc. 9th Conference on Electronic Computation, ASCE, pp 471 - 482.

11 Deb, K. (1991) "Optimal design of welded beam structure via genetic algorithms", AIAA Journal, 29, (II) pp 2013 - 2051.

12 Rajeev, S. and Krishnamoorthy, C. S. (1992) "Discrete optimization of structures using genetic algorithm", Journal of Structural Engineering, ASCE, 118, 5, pp 1233-1250

13 Reddy, J. N. (1993) "An Introduction to Finite Element Methods", McGraw - Hill, Singapore.

14 Hetenyi, M. (1946) "Beams on Elastic Foundation", The University of Michigan Press.

15 Deb, K. (1995) "Optimization for Engineering Design-Algorithms and Design", Prentice-Hall of India, New Delhi.